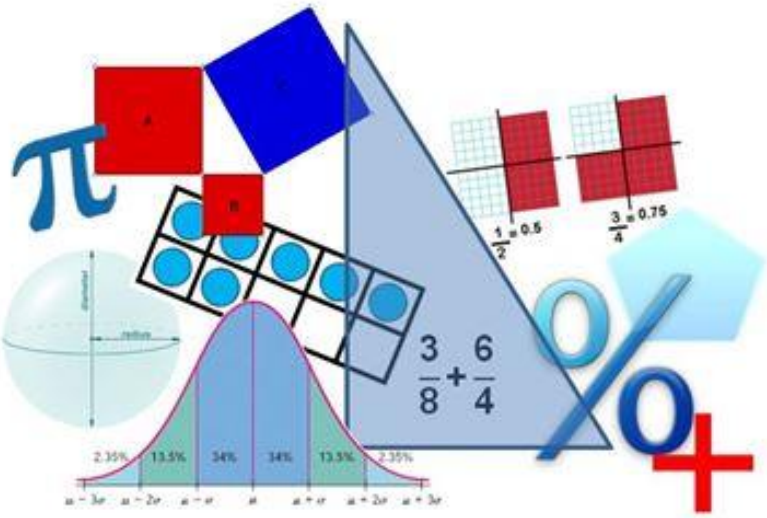
Mathematics

2016 Standards of Learning

Grade 2

Curriculum Framework



Board of Education
Commonwealth of Virginia

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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade 3 state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Students in kindergarten through grade two have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem-solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

2.1 The student will

a) read, write, and identify the place and value of each digit in a three-digit numeral, with and without models;

b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;

c) compare and order whole numbers between 0 and 999; and

d) round two-digit numbers to the nearest ten.

Understanding the Standard	Essential Knowledge and Skills
• The number system is based on a simple pattern of tens where each place has ten times the value of the place to its right. • Numbers are written to show how many hundreds, tens, and ones are in the number. • Opportunities to experience the relationships among hundreds, tens, and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a three-digit number. This structure is helpful when comparing and ordering numbers. • Manipulatives that can be physically connected and separated into groups of tens and leftover ones (e.g., snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, beads on pipe cleaners, etc.) should be used. • Ten-to-one trading activities with manipulatives on place value mats provide experiences for developing the understanding of the places in the base-10 system. • Models that clearly illustrate the relationships among ones, tens, and hundreds, are physically proportional (e.g., the tens piece is ten times larger than the ones piece). • Flexibility in thinking about numbers is critical (e.g., 84 is equivalent to 8 tens and 4 ones, or 7 tens and 14 ones, or 5 tens and 34 ones, etc.). This flexibility builds background understanding for the ideas used when regrouping. When subtracting 18 from 174, a student may choose to regroup and think of 174 as 1 hundred, 6 tens, and 14 ones. • Hundreds charts can serve as helpful tools as students develop an understanding of 10 more, 10 less, 100 more and 100 less. • Rounding a number to the nearest ten means determining which two tens the number lies between and then which ten the number is closest to (e.g., 48 is between 40 and 50 and rounded to the nearest ten is 50, because 48 is closer to 50 than it is to 40).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate understanding of the ten-to-one relationships among ones, tens, and hundreds, using manipulatives. (a) • Write numerals, using a model or pictorial representation (i.e., a picture of base-10 blocks). (a) • Read three-digit numbers when shown a numeral, a model of the number, or a pictorial representation of the number. (a) • Identify and write the place (ones, tens, hundreds) of each digit in a three-digit numeral. (a) • Determine the value of each digit in a three-digit numeral (e.g., in 352, the 5 represents 5 tens and its value is 50). (a) • Use models to represent numbers in multiple ways, according to place value (e.g., 256 can be 1 hundred, 14 tens, and 16 ones, 25 tens and 6 ones, etc.). (a) • Use place value understanding to identify the number that is 10 more, 10 less, 100 more, or 100 less than a given number, up to 999. (b) • Compare two numbers between 0 and 999 represented with concrete objects, pictorially or symbolically, using the symbols (>, <, or =) and the words *greater than, less than* or *equal to*. (c)

2.1 The student will

a) read, write, and identify the place and value of each digit in a three-digit numeral, with and without models;

b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;

c) compare and order whole numbers between 0 and 999; and

d) round two-digit numbers to the nearest ten.

Understanding the Standard	Essential Knowledge and Skills
• Rounding is an estimation strategy that is often used to assess the reasonableness of a solution or to give an estimate of an amount. • Vertical and horizontal number lines are useful tools for developing the concept of rounding. Rounding to the nearest ten using a number line is done as follows: – Locate the number on the number line. Identify the two closest tens the number comes between. – Determine the closest ten. If the number in the ones place is 5 (halfway between the two tens), round the number to the higher ten. • Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*.	• Order three whole numbers between 0 and 999 represented with concrete objects, pictorially, or symbolically from least to greatest and greatest to least. (c) • Round two-digit numbers to the nearest ten. (d)

2.2 The student will

a) count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10;

b) count backward by tens from 120; and

c) use objects to determine whether a number is even or odd.

Understanding the Standard	Essential Knowledge and Skills
• Collections of objects can be grouped and skip counting can be used to count the collection. • The patterns developed as a result of grouping and/or skip counting are precursors for recognizing numeric patterns, functional relationships, concepts underlying money, and telling time. Powerful models for developing these concepts include counters, number charts (e.g., hundreds charts, 120 charts, 200 charts, etc.) and calculators. • Skip counting by fives lays the foundation for reading a clock to the nearest five minutes and counting nickels. • Skip counting by tens lays the foundation for use of place value and counting dimes. • Calculators can be used to display the numeric patterns resulting from skip counting. Use the constant feature of the four-function calculator to display the numbers in the sequence when skip counting by that constant. • Odd and even numbers can be explored in different ways (e.g., dividing collections of objects into two equal groups or pairing objects). When pairing objects, the number of objects is even when each object has a pair or partner. When an object is left over, or does not have a pair, then the number is odd.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine patterns created by counting by twos, fives, and tens to 120 on number charts. (a) • Describe patterns in skip counting and use those patterns to predict the next number in the counting sequence. (a) • Skip count by twos, fives, and tens to 120 from various multiples of 2, 5 or 10, using manipulatives, a hundred chart, mental mathematics, a calculator, and/or paper and pencil. (a) • Skip count by two to 120 starting from any multiple of 2. (a) • Skip count by five to 120 starting at any multiple of 5. (a) • Skip count by 10 to 120 starting at any multiple of 10. (a) • Count backward by 10 from 120. (b) • Use objects to determine whether a number is even or odd (e.g., dividing collections of objects into two equal groups or pairing objects). (c)

2.3 The student will

a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and

b) write the ordinal numbers 1^{st} through 20^{th}.

Understanding the Standard	Essential Knowledge and Skills
• The cardinal and ordinal understanding of numbers is necessary to quantify, measure, and identify the order of objects. • The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., a student's position in line when students are lined up alphabetically by first name). • The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects (e.g., from the left, right, top, bottom). • Ordinal position can also be emphasized through sequencing events (e.g., days in a month or events in a story). • Practical applications of ordinal numbers can be experienced through calendar and patterning activities.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count an ordered set of objects, using the ordinal number words *first* through *twentieth*. (a) • Identify the ordinal positions first through twentieth, using an ordered set of objects presented in lines or rows from – left to right; – right to left; – top to bottom; and – bottom to top**.** (a) • Write 1^{st}, 2^{nd}, 3^{rd}, through 20^{th} in numerals. (b)

2.4 The student will

a) name and write fractions represented by a set, region, or length model for halves, fourths, eighths, thirds, and sixths;

b) represent fractional parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models.

Understanding the Standard	Essential Knowledge and Skills
• Students need opportunities to solve practical problems involving fractions in which students themselves are determining how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to count the fractional parts and recreate the whole. • Counting unit fractional parts as they build the whole (e.g., one-fourth, two-fourths, three-fourths, and four-fourths), will support students understanding that four-fourths makes one whole and prepares them for the study of multiplying unit fractions (e.g., $4 \times \frac{1}{4}$ is $\frac{4}{4}$ or one whole) in later grades. • When working with fractions, the whole must be defined. • A fraction is a numerical way of representing part of a whole region (i.e., an area model), part of a group (i.e., a set model), or part of a length (i.e., a measurement model). • In a region/area model, the parts must have the same area. • In a set model, the set represents the whole and each item represents an equivalent part of the set. For example, in a set of six counters, one counter represents one-sixth of the set. In the set model, the set can be subdivided into subsets with the same number of items in each subset. For example, a set of six counters can be subdivided into two subsets of three counters each and each subset represents one-half of the whole set. • In the primary grades, students may benefit from experiences with sets that are comprised of congruent figures (e.g., 12 eggs in a carton) before working with sets that have noncongruent parts. • In a length model, each length represents an equal part of the whole. For example, given a strip of paper, students could fold the strip into four equal parts, each part representing one-fourth. Students will notice that there are four one-fourths in the entire length of the strip of paper that has been divided into fourths. • Students need opportunities to use models (region/area or length/measurement) to count fractional parts that go beyond one whole. For instance, if students are counting five pie pieces	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize fractions as representing equal-size parts of a whole. (a) • Name and write fractions represented by a set model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Name and write fractions represented by a region/area model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Name and write fractions represented by a length model showing halves, fourths, eighths, thirds, and sixths. (a, b) • Represent, with models and with symbols, fractional parts of a whole for halves, fourths, eighths, thirds, and sixths, using: – region/area models (e.g., pie pieces, pattern blocks, geoboards); – sets (e.g., chips, counters, cubes); and – length/measurement models (e.g., fraction strips or bars, rods, connecting cube trains). (b) • Compare unit fractions for halves, fourths, eighths, thirds, and sixths), using words (greater than, less than or equal to) and symbols (>, <, =), with models. (c) • Using same-size fraction pieces, from region/area models or length/measurement models, count the pieces (e.g., *one-fourth, two-fourths, three-fourths*, etc.) and compare those pieces to one whole (e.g., *four-fourths* will make one whole*; one-fourth* is less than a whole). (c)

2.4 The student will

a) name and write fractions represented by a set, region, or length model for halves, fourths, eighths, thirds, and sixths;

b) represent fractional parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models.

Understanding the Standard	Essential Knowledge and Skills
and building the pie as they count, where each piece is equivalent to one-fourth of a pie, they might say "one-fourth, two-fourths, three-fourths, four-fourths, five-fourths." As a result of building the whole while they are counting, they begin to realize that four-fourths make one whole and the fifth-fourth starts another whole. They will begin to generalize that when the numerator and the denominator are the same, they have one whole. They also will begin to see a fraction as the sum of unit fractions (e.g., three-fourths contains three one-fourths or four-fourths contains four one-fourths which is equal to one whole). This provides students with a visual for when one whole is reached and develops a greater understanding of numerator and denominator. • Students will learn to write names for fractions greater than one and for mixed numbers in grade three. • Creating models that have a fractional value greater than one whole and describing those models as having a whole and leftover equal-sized pieces are the foundation for understanding mixed numbers in grade three. • When given a fractional part of a whole and its value (e.g., one-third), students should explore how many one-thirds it will take to build one whole, to build two wholes, etc. If this ☐ is $\frac{1}{3}$, then this is the whole ☐☐☐. If this is the whole ☐☐☐,then this ☐ is $\frac{1}{3}$. • Students should have experiences dividing a whole into additional parts. As the whole is divided into more parts, students understand that each part becomes smaller (e.g., folding a paper in half one time, creates two halves; folding it in half again, creates four fourths, which is smaller; folding it in half again, creates eight eighths, which is even smaller). The same concept can be applied to thirds and sixths. • The value of a fraction is dependent on both the number of equivalent parts in a whole (denominator) and the number of those parts being considered (numerator). • Students should have opportunities to make connections among fraction representations by connecting concrete or pictorial representations with spoken or symbolic representations.	

2.4 The student will

a) name and write fractions represented by a set, region, or length model for halves, fourths, eighths, thirds, and sixths;

b) represent fractional parts with models and with symbols; and

c) compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models.

Understanding the Standard	Essential Knowledge and Skills
• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions will further this development (e.g., *thirds* means "three equal parts of a whole" or $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students). • A unit fraction is when there is a one as the numerator. • Using models when comparing unit fractions builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (i.e., the larger the denominator the smaller the piece; therefore, $\frac{1}{3} > \frac{1}{4}$).	

A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have if Jackie gives us five more?" or "About how many students are at two tables?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In kindergarten through grade two, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition and subtraction within 20;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable; and
- experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects as well as problems that involve sharing equally, the initial work with fractions.

2.5 The student will

a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and

b) demonstrate fluency with addition and subtraction within 20.

Understanding the Standard	Essential Knowledge and Skills
• Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Concrete models should be used initially to develop an understanding of addition and subtraction facts. • Recognizing and using patterns and learning to represent situations mathematically are important aspects of primary mathematics. • An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). • Equations may be written with sums and differences at the beginning of the equation (e.g., 8 = 5 + 3). • An equation can be represented using balance scales, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal sign (e.g., 5, 4 + 3, 8 - 2). It is not necessary for students at this level to use the term 'expression.' • The patterns formed by related facts facilitate the solution of problems involving a missing addend in an addition sentence or a missing part in a subtraction sentence. • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the addition and subtraction facts include: – counting on; – counting back; – "one more than," "two more than";	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and use the relationship between addition and subtraction to solve single-step practical problems, with whole numbers to 20. (a) • Determine the missing number in an equation (number sentence) (e.g., 3 + ☐ = 5 or ☐ + 2 = 5; 5 – ☐ = 3 or 5 – 2 = ☐). (a) • Write the related facts for a given addition or subtraction fact (e.g., given 3 + 4 = 7, write 7 – 4 = 3 and 7 – 3 = 4). (a) • Demonstrate fluency with addition and subtraction within 20. (b)

2.5 The student will

a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and

b) demonstrate fluency with addition and subtraction within 20.

Understanding the Standard	Essential Knowledge and Skills
– "one less than," "two less than"; – "doubles" (e.g., 2 + 2 = ☐; 3 + 3 = ☐); – "near doubles" (e.g., 3 + 4 = (3 + 3) + 1 =☐); – "make 10" facts (7 + 4 can be thought of as 7 + 3 + 1 in order to make a 10); – "think addition for subtraction," (e.g., for 9 – 5 = ☐, think "5 and what number makes 9?"); – use of the commutative property (e.g., 4 + 3 is the same as 3 + 4); – use of related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); – use of the additive identity property (e.g., 4 + 0 = 4); and – use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.) • Grade two students should begin to explore the properties of addition as strategies for solving addition and subtraction problems using a variety of representations. • The properties of the operations are "rules" about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level: – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). – The identity property of addition states that if zero is added to a given number, the sum is the same as the given number (e.g., 0 + 2 = 2). – The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., 4 + (6 + 7) = (4 + 6) + 7). • Addition and subtraction problems should be presented in both horizontal and vertical written format. • Models such as 10 or 20 frames and part-part-whole diagrams help develop an understanding of relationships between equations and operations.	

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills
• Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Grade two students should begin to explore the properties of addition as strategies for solving addition and subtraction problems using a variety of representations, including manipulatives and diagrams. • The properties of the operations are "rules" about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level: – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). – The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. – The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., 4 + (6 + 7) = (4 + 6) + 7). • An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3, and 4 + 3 = 9 - 2). An equation can be represented using a balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • Rounding is one strategy used to estimate. • Estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. • Estimation can be used to check the reasonableness of the sum or difference when an exact answer is required.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate the sum of two whole numbers whose sum is 99 or less and recognize whether the estimation is reasonable (e.g., 27 + 41 is about 70, because 27 is about 30 and 41 is about 40, and 30 + 40 is 70). (a) • Estimate the difference between two whole numbers each 99 or less and recognize whether the estimate is reasonable. (a) • Determine the sum of two whole numbers whose sum is 99 or less, using various methods. (b) • Determine the difference of two whole numbers each 99 or less, using various methods. (b) • Create and solve single-step practical problems involving addition or subtraction. (c) • Create and solve two-step practical problems involving addition, subtraction, or both addition and subtraction. (c)

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills
• Problem solving means engaging in a task for which a solution or a method of solution is not known in advance. Solving problems using data and graphs offers one way to connect mathematics to practical situations. • The problem-solving process is enhanced when students: – create their own story problems; and – model word problems, using manipulatives, drawings, or acting out the problem. • The least number of steps necessary to solve a single-step problem is one. • Using concrete materials (e.g., base-10 blocks, connecting cubes, beans and cups, etc.) to explore, model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. • Conceptual understanding begins with concrete and contextual experiences. Next, students must make connections that serve as a bridge to the symbolic. Student-created representations, such as drawings, diagrams, tally marks, graphs, or written comments are strategies that help students make these connections. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses. • Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. • Students should experience a variety of problem types related to addition and subtraction. Problem type examples are included in the following chart:	

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills

GRADE 2: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES

Join (Result Unknown)	Join (Change Unknown)	Join (Start Unknown)
Sue had 28 pencils. Alex gave her 14 more pencils. How many pencils does Sue have all together?	Sue had 28 pencils. Alex gave her some more pencils. Now Sue has 42 pencils. How many did Alex give her?	Sue had some pencils. Alex gave her 14 more. Now Sue has 42 pencils. How many pencils did Sue have to start with?
Separate (Result Unknown)	**Separate (Change Unknown)**	**Separate (Start Unknown)**
Brooke had 35 marbles. She gave 19 marbles to Joe. How many marbles does Brooke have now?	Brooke had 35 marbles. She gave some to Joe. She has 16 marbles left. How many marbles did Brooke give to Joe?	Brooke had some marbles. She gave 19 to Joe. Now she has 16 marbles left. How many marbles did Brooke start with?
Part-Part-Whole (Whole Unknown)	**Part-Part-Whole (One Part Unknown)**	**Part-Part-Whole (Both Parts Unknown)**
The teacher has 20 red markers and 25 blue markers. How many markers does he have?	The teacher has 45 markers. Twenty of the markers are red, and the rest are blue. How many blue markers does he have?	The teacher has a tub of red and blue markers. She has 45 markers in all. How many markers could be red? How many could be blue?
Compare (Difference Unknown)	**Compare (Bigger Unknown)**	**Compare (Smaller Unknown)**
Ryan has 20 books and Chris has 9 books. How many more books does Ryan have than Chris? Ryan has 20 books. Chris has 9 books. How many fewer books does Chris have than Ryan?	Chris has 9 books. Ryan has 11 more books than Chris. How many books does Ryan have? Chris has 11 fewer books than Ryan. Chris has 9 books. How many books does Ryan have?	Ryan has 11 more books than Chris. Ryan has 20 books. How many books does Chris have? Chris has 11 fewer books than Ryan. Ryan has 20 books. How many books does Chris have?

2.6 The student will

a) estimate sums and differences;

b) determine sums and differences, using various methods; and

c) create and solve single-step and two-step practical problems involving addition and subtraction.

Understanding the Standard	Essential Knowledge and Skills
• Strategies for adding and subtracting two-digit numbers can include, but are not limited to, using concrete objects, a hundred chart, number line, and invented strategies. • Mental computation helps build number sense in students. Strategies for mentally adding or subtracting two-digit numbers should be student-invented strategies. Some of these strategies may include: *Partial Sums* 56 + 41 = __ 50 + 40 = 90 6 + 1 = 7 90 + 7 = 97 *Counting On* 36 + 62 = __ 36 + 60 = 96 96 + 2 = 98 87 – 25 = __ 20 + 60 = 80 5 + 2 = 7 60 + 2 = 62 87 – 25 = __ 25 + 60 = 85 85 + 2 = 87 60 + 2 = 62 87 – 25 = __ 25 + 2 = 27 27 + 60 = 87 2 + 60 = 62 98 – 41 = __ 98 – 40 = 58 58 – 1 = 57 56 – 21 = __ 50 – 20 = 30 6 – 1 = 5 30 + 5 = 35 • The terms used in addition are 23 → *addend* + 46 → *addend* 69 → *sum* • The terms often used in subtraction are 98 → *minuend* – 41 → *subtrahend* 57 → *difference* • At this level, students do not need to use the terms *addend, minuend, or subtrahend* for addition and subtraction as shown above.	

The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in kindergarten through grade two encounter measurement in their daily lives, from their use of the calendar and science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight, capacity, time, temperature, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain a deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply several techniques, direct comparison, nonstandard units, and standard tools to determine measurements and to develop an understanding of the use of U.S. Customary units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three).
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

2.7 The student will

a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and

b) use the cent symbol, dollar symbol, and decimal point to write a value of money.

Understanding the Standard	Essential Knowledge and Skills
• The money system used in the United States consists of coins and bills based on relationships involving ones, fives, and tens. The dollar is the basic unit. • The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills into groups that are easier to count. • Simulate everyday opportunities to count and compare a collection of coins and one-dollar bills whose total value is $2.00 or less. • Emphasis is placed on the verbal expression of the symbols for cents and dollars (e.g., $0.35 and 35¢ are both read as "thirty-five cents"; $2.00 is read as "two dollars").	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the value of a collection of coins and one-dollar bills whose total value is $2.00 or less. (a) • Count by ones, fives, tens, and twenty-fives to determine the value of a collection of coins whose total value is $2.00 or less. (a) • Compare the values of two sets of coins and one-dollar bills (each set having a total value of $2.00 or less), using the terms *greater than, less than*, or *equal to*. (a) • Use the cent (¢) and dollar ($) symbols and decimal point (.) to write a value of money which is $2.00 or less. (b)

2.8 The student will estimate and measure

a) length to the nearest inch; and

b) weight to the nearest pound.

Understanding the Standard	Essential Knowledge and Skills
• The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units. • Measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book; the weight of the cube measures the weight of the book). • A clear concept of the size of one unit is necessary before one can measure to the nearest unit. • The experience of making a ruler out of individual units of length can lead to greater understanding of using one. A ruler takes those units of length and numbers them. Measurement of length is counting the number of units. A "broken ruler" is a useful tool for students to use in order to develop an understanding of counting the number of units. • Students benefit from experiences that allow them to explore the relationship between the size of the unit of measurement and the number of units needed to measure the length of an object. • Benchmarks of common objects need to be established for one pound. Practical experience measuring the weight of familiar objects helps to establish benchmarks.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a ruler as an instrument to measure length. (a) • Estimate and then measure the length of various line segments and objects to the nearest inch using a ruler. (a) • Identify different types of scales as instruments to measure weight. (b) • Estimate and then measure the weight of objects to the nearest pound using a scale. (b)

2.9 The student will tell time and write time to the nearest five minutes, using analog and digital clocks.

Understanding the Standard	Essential Knowledge and Skills
• Telling time requires reading a clock. The position of the two hands on an analog clock is read to tell the time. A digital clock shows the time by displaying the time in numbers which are read as the hour and minutes. • Counting by fives is beneficial when telling time to the nearest five minutes. • Students should develop an understanding that there are 60 minutes in an hour. • The use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise at all times.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Show, tell, and write time to the nearest five minutes, using an analog and digital clock. • Match a written time (e.g., 4:20, 10:05, 1:50) to a time shown on a clock face to the nearest five minutes. • Match the time (to the nearest five minutes) shown on a clock face to a written time.

2.10 The student will

a) determine past and future days of the week; and

b) identify specific days and dates on a given calendar.

Understanding the Standard	Essential Knowledge and Skills
• The calendar is a way to represent units of time (e.g., days, weeks, months, and years). • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. • Practical situations are appropriate to develop a sense of the interval of time between events (e.g., club meetings occur every week on Monday: there is a week between meetings).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the day that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. (a) • Identify specific days and dates (e.g., What is the third Monday in a given month? What day of the week is May 11?). (b)

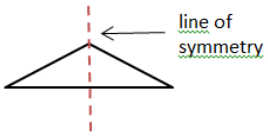
2.11 The student will read temperature to the nearest 10 degrees.

Understanding the Standard	Essential Knowledge and Skills
• The symbols for degrees in Fahrenheit (°F) should be used to write temperatures. • Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of things found in the student's environment (e.g., temperature of the classroom; temperature on the playground; temperature of warm and cold liquids; body temperature). • Estimating and measuring temperatures in the environment requires the use of real thermometers. • A variety of physical models (e.g., circular, linear) should be used to represent the temperature determined by a real thermometer. • A physical or pictorial model can be used to represent the temperature measured using a real thermometer. • Reading temperature in degrees Celsius will begin in grade three.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify different types of thermometers as instruments used to measure temperature. • Read temperature in Fahrenheit to the nearest ten degrees on thermometers (real world, physical model, and pictorial representations).

2.12 The student will

a) draw a line of symmetry in a figure; and

b) identify and create figures with at least one line of symmetry.

Understanding the Standard	Essential Knowledge and Skills
• A line of symmetry divides a figure into two congruent parts each of which is the mirror image of the other. An example is shown below:  Lines of symmetry are not limited to horizontal and vertical lines. • Children learn about symmetry through hands-on experiences with geometric figures and the creation of geometric pictures and patterns. • Guided explorations of the study of symmetry using mirrors, paper folding, and pattern blocks will enhance students' understanding of the attributes of symmetrical figures. • Congruent figures have exactly the same size and shape. Noncongruent figures do not have exactly the same size and shape. Congruent figures remain congruent even if they are in different spatial orientations. • While investigating symmetry, children move figures, such as pattern blocks, intuitively, thereby exploring transformations of those figures. A transformation is the movement of a figure — either a translation, rotation, or reflection. A translation is the result of sliding a figure in any direction; rotation is the result of turning a figure around a point or a vertex; and reflection is the result of flipping a figure over a line. Children at this level do not need to know the terms related to transformations of figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Draw a line of symmetry in a figure. (a) • Identify figures with at least one line of symmetry, using various concrete materials (e.g., mirrors, paper folding, pattern blocks). (b) • Determine a line of symmetry that results in two figures that have the same size and shape and explain reasoning. (a, b) • Create figures with at least one line of symmetry using various concrete materials. (b)

2.13 The student will identify, describe, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).

Understanding the Standard	Essential Knowledge and Skills
• A plane figure is any closed, two-dimensional shape. • A vertex is a point at which two or more lines, line segments, or rays meet to form an angle. In solid figures a vertex is the point at which three or more edges meet. • An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect. • A solid figure is a three-dimensional figure, having length, width, and height. • A circle is the set of points in a plane that are the same distance from a point called the center. • A sphere is a solid figure with all of its points the same distance from its center. • A rectangle is a quadrilateral with four right angles. A square is a special type of rectangle. • A square is a quadrilateral with four congruent (equal length) sides and four right angles. • A right angle measures exactly 90 degrees. • A rectangular prism is a solid figure in which all six faces are rectangles. A rectangular prism has eight vertices and 12 edges. • A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has eight vertices and 12 edges. It is a type of rectangular prism. • The edge is the line segment where two faces of a solid figure intersect. • A face is any flat side of a solid figure (e.g., a square is a face of a cube). • Tracing faces of cubes and rectangular prisms and decomposing cubes and rectangular prisms along their edges helps students understand the set of plane figures related to the solid figure. • The relationship between plane and solid figures, such as the square and the cube or the rectangle and the rectangular prism helps build the foundation for future geometric study of faces, edges, angles, and vertices. The following chart defines the characteristics of solid figures included at this grade level:	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine similarities and differences between related plane and solid figures (circles/spheres, squares/cubes, rectangles/rectangular prisms), using models and cutouts. • Trace faces of solid figures (cubes and rectangular prisms) to create the set of plane figures related to the solid figure. • Identify and describe plane figures (circles, squares, and rectangles), according to their characteristics (number of sides, vertices, and angles). Squares and rectangles have four right angles. • Identify and describe solid figures (spheres, cubes, and rectangular prisms), according to the shape of their faces, number of edges, and number of vertices, using models. • Compare and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms) according to their characteristics (number and shape of their faces, edges, vertices, and angles).

2.13 The student will identify, describe, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).

Understanding the Standard	Essential Knowledge and Skills

Solid Figure	# of Faces	Shape of Faces	# of Edges	# of Vertices
Cube	6	Squares	12	8
Rectangular Prism	6	Rectangles	12	8
Sphere	0	N/A	0	0

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. In grade two, students experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of which events are more or less likely to occur.

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

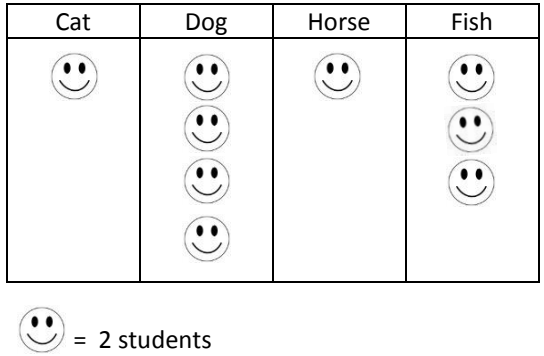
2.14 The student will use data from probability experiments to predict outcomes when the experiment is repeated.

Understanding the Standard	Essential Knowledge and Skills
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in investigations and have opportunities to use manipulatives. • Investigation of experimental probability is continued through informal activities, such as dropping a two-colored counter (usually a chip that has a different color on each side), using a multicolored spinner (a circular spinner that is divided equally into two, three, four, six or eight parts where each part is filled with a different color), using spinners with numbers, or rolling random number cubes. • Probability is the chance of an event occurring. • An event is a possible outcome in probability. Simple events include the possible outcomes when tossing a coin (heads or tails), when rolling a random number cube or when spinning a spinner.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Conduct probability experiments using multicolored spinners, colored tiles, or number cubes and use the data from the experiments to predict outcomes if the experiment is repeated. • Record the results of probability experiments, using tables, charts, and tally marks. • Interpret the results of probability experiments. • Predict which of two events is more or less likely to occur if an experiment is repeated.

2.15 The student will

a) collect, organize, and represent data in pictographs and bar graphs; and

b) read and interpret data represented in pictographs and bar graphs.

Understanding the Standard	Essential Knowledge and Skills
• Data can be collected and organized in pictographs and bar graphs. • The purpose of a graph is to represent data gathered to answer a question. • At this level, the number of categories on a pictograph should be limited to four when a student is creating a graph, and six when a student is interpreting and analyzing a graph. • A pictograph uses pictures or symbols to represent one or more objects (data points). • A key is provided in a graph to assist in the analysis of the displayed data. • The key should be provided for the symbol in a pictograph graph when the symbol represents more than one piece of data (e.g., represents five people in a graph). At this level, each symbol should represent 1, 2, 5, or 10 pieces of data. • An example of a pictograph is: The Types of Pets We Have  = 2 students • In prior grades, students worked with simple pictographs with a scale of one (i.e., each picture represented only one item) making a key unnecessary.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect and organize data using various forms of data collection (e.g., lists, tables, objects, pictures, symbols, tally marks, charts). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a) • Represent data in pictographs and bar graphs (limited to 16 or fewer data points for no more than four categories). (a) • Read and interpret data represented in pictographs and bar graphs with up to 25 data points for no more than six categories (represented horizontally or vertically). State orally and in writing (at least one statement) that includes one or more of the following: – Describes the categories of data and the data as a whole (e.g., adding together all data points will equal the total number of responses); – Identifies parts of the data that have special characteristics; including categories with the greatest, the least, or the same; – Uses the data to make comparisons; and – Makes predictions and generalizations. (b)

2.15 The student will

a) collect, organize, and represent data in pictographs and bar graphs; and

b) read and interpret data represented in pictographs and bar graphs.

Understanding the Standard	Essential Knowledge and Skills
• Students' prior knowledge and work with skip counting helps them to identify the number of pictures or symbols to be used in a pictograph. • Definitions for the terms picture graph and pictographs vary. Pictographs are most often defined as a pictorial representation of numerical data. The focus of instruction should be placed on reading and using the key in analyzing the graph. There is no need for students to distinguish between a picture graph and a pictograph. • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper may ensure more accurate graphs. – A bar graph uses horizontal or vertical parallel bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after each of the bars. – The axis displaying the scale that represents the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. In grade two, students should collect data that are recorded in increments of whole numbers limited to multiples of 1, 2, or 5. – At this level, the number of categories on a bar graph should be limited to four. A key should be included where appropriate. – Each axis should be labeled, and the graph should be given a title. • Statements that represent an analysis and interpretation of the data in the graph should be discussed with students and written (e.g., similarities and differences, least and greatest, the categories, total number of responses, etc.). • Data gathered and displayed by students should be limited to 16 or fewer data points for no more than four categories. However, students at this level should be able to interpret graphs that contain data points that represent their entire class (e.g., approximately 25 data points).	

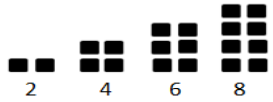
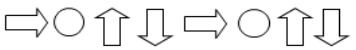
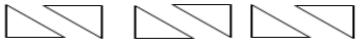
Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through grade two develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

2.16 The student will identify, describe, create, extend, and transfer patterns found in objects, pictures, and numbers.

Understanding the Standard	Essential Knowledge and Skills
• Patterning is a fundamental cornerstone of mathematics, particularly algebra. The process of generalization leads to the foundation of algebraic reasoning. • Opportunities to create, identify, describe, extend, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically. • The part of the pattern that repeats is called the core. • Growing patterns involve a progression from step to step which make them more difficult for students than repeating patterns. Students must determine what comes next and also begin the process of generalization, which leads to the foundation of algebraic reasoning. Students need experiences identifying what changes and what stays the same in a growing pattern. Growing patterns may be represented in various ways, including dot patterns, staircases, pictures, etc. • In numeric patterns, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. Students do not need to use the term *common difference* at this level. • Sample numeric patterns include: – 6, 9, 12, 15, 18, …(growing pattern); – 2, 4, 6, 8, 10, … (growing pattern); and – 1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern). In grade two, growing numeric patterns will only include increasing values. • In patterns using objects or figures, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure, and reflection is the result of flipping a figure over a line. • Examples of patterns using objects or figures include: –  –  – 	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify a pattern as growing or repeating. • Describe the core (the part of the sequence that repeats) of a given repeating pattern. • Describe how a given growing pattern is changing. • Create a growing or repeating pattern, using objects, pictures, or numbers. • Extend a given pattern, using objects, pictures, or numbers. • Transfer a given growing or repeating pattern from one form to another using objects, pictures, or numbers.

2.16 The student will identify, describe, create, extend, and transfer patterns found in objects, pictures, and numbers.

Understanding the Standard	Essential Knowledge and Skills
• Transferring a pattern is creating the pattern in a different form or representation. • Examples of pattern transfers include: – 10, 20, 30, 40 has the same structure as 14, 24, 34, 44; – has the same structure as ; and – 1, 3, 5, 1, 3, 5, 1, 3, 5 has the same structure as ABCABC.	

2.17 The student will demonstrate an understanding of equality through the use of the equal symbol and the use of the not equal symbol.

Understanding the Standard	Essential Knowledge and Skills
• The equal symbol (=) means that the values on either side are equivalent (balanced). • The not equal (≠) symbol means that the values on either side are not equivalent (not balanced). • In order for students to develop the concept of equality, students need to see the = symbol used in various appropriate locations (e.g., 3 + 4 = 7 and 5 = 2 + 3). • An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an 'equal' symbol (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 - 2). An equation can be represented using a number balance scale, with equal amounts on each side (e.g., 3 + 5 = 6 + 2). • An expression represents a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., 5, 4 + 3, 8-2). Students at this level are not expected to use the terms expression or variable. • Manipulatives such as connecting cubes, counters, and number scales can be used to model equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the equal symbol (=) as the symbol used to indicate that the values on either side are equal. • Identify the not equal symbol (≠) as the symbol used to indicate that two values on either side are not equal. • Identify values and expressions that are equal (e.g., 8 = 8, 8 = 4 + 4). • Identify values and expressions that are not equal (e.g., 8 ≠ 9, 4 + 3 ≠ 8). • Identify and use the appropriate symbol to distinguish between equal and not equal quantities (e.g., 9 + 24 = 10 + 23; 45 – 9 = 46 – 10; 15 + 16 ≠ 31 + 15). • Use a model to represent the relationship of two expressions of equal value and two expressions that are not equivalent.